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VARIATIVE MODELLING OF ONE-DIMENSIONAL CONTOURS ACCORDING TO THE GIVEN CONDITIONS

Model of the surface of complex shape is usually formed on the basis of the frame, elements of which are plane and spatial curves. The task of ensuring the functional properties of these surfaces requires the development of methods for the formation of linear elements of the model as a one-dimensional contours with the desirable law of geometric characteristics change. The order of fixation and order of smoothness of contour, curvature and torsion values which change along the curve are those characteristics for spatial curves.

Variative discrete geometric modeling involves the definition of the surface using an initial points array and linear elements of the model - with the using of a points of the sets. Curve, which is represented by an ordered set of points will be called a discretely represented of the curve, or DRC. Differential-geometric characteristics of the DRC are also included in the determinant. These characteristics are provided during simulation. DRC is formed by discrete interpolation or thickening, which involves determining the position of the intermediate points for the initial points set [2].

During the simulation of differential-geometric characteristics of the curve will be evaluated by their discrete counterparts, which will be called the discrete characteristics.

The chord of accompanying a broken line (ABL) will be regarded as an approximate position of the tangent line. The discrete characteristic which corresponds to the osculating plane, is the plane which passes through three successive points of the DRC. This plane will be called the adjoining. The radius of curvature at the DRC will be evaluated using the radius of the circle adjoining, which is determined by this point, the previous and subsequent points of the points set. As discrete torsion will use the value of the relationship of the angle between the adjacent planes to the length of the corresponding chord of ABL.

The necessary condition of modelling of differential-geometric characteristics of contour is a regular change in the values of the discrete characteristics and existence of the limit to which their values approaching in the process of successive condensations. The algorithm of thickening provides the specified dynamics of the discrete characteristics change. Thus, a discrete geometric model of the curve is the points set, discrete characteristics and the algorithm of thickening.

The second order of smoothness and regular change of curvature and torsion values are main characteristics of contours generated by our methods. Under the DRC of second order of smoothness we mean a curved line, which is represented by a points set, for which the algorithm of thickening performs the following conditions.

1. In the process of successive steps of modelling the angle between the chords of ABL, which are restricted by the point *i* of KDP and the nearest preceding and following points approaches to zero.

2 The adjacent planes, one of which determined by the point *i* and the two nearest points of the previous set, and the other – by point *i* and two points closest subsequent rows in the successive modeling tend to occupy the position of the *i*-th adjacent the plane defined by point *i*, the nearest previous and subsequent points.

3. Value of the ratio the angle between the the *i*-th and previous adjacent planes to chord ABL, which is located on the line of intersection of these planes, tends to the

same value, defined the *i*-th and subsequent adjacent planes.

4 The radiuses of three circles which pass through the point *i* and the next two previous, next, previous and subsequent points of a set approach to the same value.

As a result of successive modelling, in the limit, we obtain a continuous one-parameter set of points, each point of which the conditions 1 and 2 provides a unique position of the main trihedral and the conditions 3 and 4, provides a unique value of torsion and curvature.

The spatial DRC is formed by thickening of an ordered points set. Every three consecutive initial points define a plane (Fig. 1).

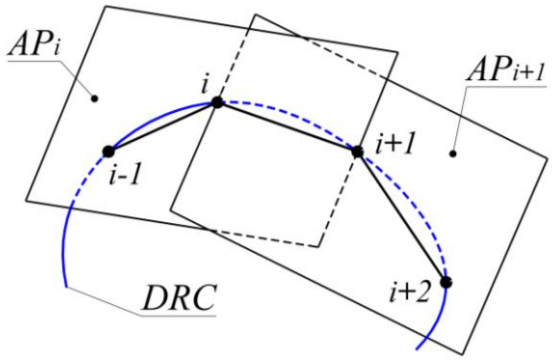

Fig. 1. The location of adjacent planes

We shall call these planes adjoining (AP*i* - plane that passes through the points *i-1*, *i*, *i + 1*). Require that value of the dihedral angle, which is limited by adjacent AP (φ_i), within which the site is located DRC does not exceed 180°. Then the direction of rotation of AP*i* an angle φ_i around the line of intersection, followed by AP*i+1* (line (*i*, *i + 1*)), in which AP*i* and AP*i*+1 coincide corresponds to the direction of turn DRC. DPC is divided into sections of right and left turn and is formed separately over these sections.

Tangents to the DRC permanent turn at the initial points (t_i) are located within two adjacent dihedral angles φ_{i-1} and φ_i simultaneously (Fig. 2).

Tangent to the DRC t_i determines the position of the planes that are tangent to the DRC at the point *i* and passing through the adjacent initial points - *i-1* and *i + 1*.

The planes which tangent to the DRC in neighboring initial points form a dihedral angle ψ_i which is the area of the location of the DRC continuous turn.

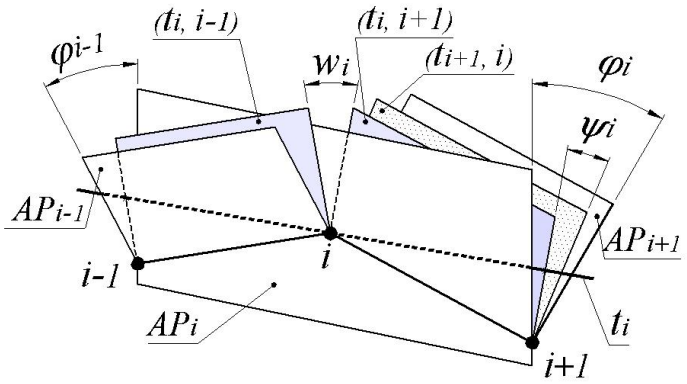

Fig. 2. Determining the position of the tangent lines and planes

The planes tangent to the PDK at one point restrict the area of possible location of the osculating plane at the point i DRC (OP*i*).

The osculating and the tangent planes at two neighboring points limit tetrahedron.

This tetrahedron is an area of possible locations DRC continuous turn with the given the main in the starting points of the trihedral. Thickening point (i_{sg}) is assigned inside the tetrahedron location DRC. For her, within the respective bands are assigned position of the tangent line and the osculating plane. As a result, each segment obtain two new tetrahedron.

The determination of positions of the tangent lines, adjoining planes and thickening points within the respective ranges provides the first order of smoothness formed contour.

At formation of contours of the second order smoothness values of torsion and curvature radiuses are evaluated using their discrete counterparts.

The radius of curvature in the *i*-th point of DRC estimated using the radii adjacent and tangential circles. The *i*-th adjacent circle defined by the point *i* and the nearest preceding and following points of DRC. The *i*-th tangent circle defined by the point *i*, the tangent line in this point and the nearest point of the DRC.

The average value of torsion on the *i*- th section of DRC, for which the position of the tangent lines are defined in the points can be estimated by the value:

$$B_i^{\varphi} = \frac{\psi_i}{h_i},$$

where $h_i = |i;i+1|$ – length of the chord of accompanying a broken line.

Tangent planes determining the value of discrete torsion and the radius of tangent circles at the points of the DRC. Their position is assigned on the basis of the conditions of regularity of the change of discrete characteristics.

Developed algorithms provide availability of ranges of possible according to the given data values of the curvature and torsion in the points of DRC and consistent convergence of these ranges to a single value.

The algorithm variative discrete geometric modeling of a smooth curve is proposed as a result of this research. The analysis of the initial points set, as a result of which is determined area of the possible location of the curve and the range of possible values of its geometrical characteristics is the basis for the algorithm. Assigned characteristics specifying the of the region of location of the curve. Values of the characteristics are provided during of successive thickening of points set.

The DRC can be formed on the basis of an arbitrary points set. At the same time it is possible to control and correction of the obtained solution on each step of the simulation, overlay additional conditions to the curve, and is guaranteed by the absence of oscillations.

Further development of methods of variative discrete geometrical modeling aimed at improving the their versatility and capacities to adapt under the requirements of concrete applied problems. This problem can be solved by increasing the number of conditions that are imposed on the contour by increasing the number of parameters forming.

Practical implementation of involves the development of methods, based on them, the software modules that are compatible with existing CAD-packages, such as COMPASS, Solid Works, AutoCAD.

Literature:

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2. V.M. Naydysh Discrete interpolation: textbook allowance. - Melitopol: Lux, 2008. – 250 pp.