Mathematical modeling of groundwaters pressure distribution in the underground structures by cylindrical form zone

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Abstract. A mathematical model is developed for studying the distribution of groundwater pressure and its variation in the zone of underground structures of a cylindrical shape. Based on the created model, the influence of the thickness of the aquifer, the soil porosity, the filtration coefficient, the viscosity coefficient and the piezoelectric conductivity coefficient on the pressure that groundwater exerts on the lower part of the underground structure is investigated. The analysis of the possibility of pushing the structure and breaking the foundation under the influence of pressure caused by groundwater is analyzed. Analytical formulas are obtained for estimating the stresses in the foundation and predicting the possibility of its destruction.

1 Problem statement

The action of groundwater on the underground parts of building structures can result in severe consequences. Underflooding of buildings causes a change in the properties of materials, and in some cases water pressure in aquifers results in deformation and breaking of structures. Therefore, a study of water pressure distribution in the zone of underground structures has a practical interest. . The initial data for carrying out calculations differ by a wide spread [1, 2], which complicates the theoretical analysis. Nevertheless the development of mathematical models based on the Darcy formula [3] can facilitate to a great extent predicting the situation connected with possible breaking of building structures in the presence of underground waters. As a rule, they do not have waterproof and are situated in the first aquifer [15].Interstate water is between two layers lower than other underground water.

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The purpose of this paper is to develop a mathematical model for analysing the distribution of groundwater along the perimeter of the underground part of the structure and to estimate the magnitude of stresses occurring in the lower part of the foundation of a cylindrical shape.

2 Water transport equations in porous soils

The fluid filtration rate is well described by the Darcy formula [3]

$$w = -k_{\phi} \frac{dH}{dx}\text{, m/s,} \tag{1}$$

where $-k_f$- filtration coefficient which is equal to the filtration rate in m/s, measured for a single water pressure gradient $dH/dx=1$

In the study of water permeability of soils, an analog of the formula is often used, when the flow velocity of the liquid is associated with the pressure gradient [3]:

$$w = -\frac{k}{\mu}\frac{dp}{dx} \tag{2}$$

where k- permeability coefficient in m^2, μ -dynamic viscosity coefficient of a fluid in Pa·s.

The pressure P caused by the height of the water column H is equal to the product of the density of water p by the acceleration due to gravity g, therefore the relation between the filtration coefficient and permeability coefficient, taking into account expressions (1) and (2), can be represented by the formula

$$k = k_f/\rho g \tag{3}$$

In practice, the kinematic viscosity of the fluid $\eta = \mu/\rho$is often used, where p is the density of the liquid in kg/m^3. The coefficient n is measured in units of m^2/s.

Water-permeable soils consist of large fragment rocks, pebble stone, gravel, sands, fractured rocks, etc. Waterproof soils include massive crystalline rocks (granite, marble) and dense clays. Semi-permeable rocks include clay sands, loose sandstones, and loose marls. The filtration coefficient value k_f of various soils [4] is given in the Table 1.

Table 1. Values k_f for different soils [1].

Soil type	Sand	Sandy loams	Clay loam	Clay
k_f, m/s	10^{-6}-10^{-3}	10^{-8}-10^{-5}	10^{-10}-10^{-7}	10^{-11}-10^{-9}

As can be seen from Table 2, the coefficient k_f varies over a wide range, even for one type of soil. We consider the movement of water in an aquifer with a thickness h, where the rate of filtration is described by formula (2). The equation of the rate of pressure redistribution in the aquifer has the form [5]:

$$\frac{\partial p}{\partial t} = \chi \Delta p(x, y, z) \tag{4}$$

where X- the pressure conductivity coefficient, sometimes called the piezoelectric conductivity coefficient, m^2/s.

To determine the conductivity coefficient, the Shchelkachev's formula is used [6]:

$$\chi = \frac{k}{\mu(m\beta_w + \beta_{gr})} = \frac{k_\phi}{\rho g(m\beta_w + \beta_{gr})} \tag{5}$$

where *m*- porosity, β_w- the coefficient of water volume compression, β_{gr}- the coefficient of volume compression of the solid part of the soil, 1/Pa.

As the value β_w = 4,9*10^{-10} m^2/n [7], aβ_{gr} = 10^{-11} - 10^{-12} m^2/n [8], formula (5) can be simplified:

$$\chi = \frac{k}{\mu m\beta_w} = \frac{k_f}{\rho g m\beta_w} \tag{6}$$

The piezoelectric conductivity coefficients of soils ***X*** calculated by formula (6) for β_w = 4,9*10^{-10} m^2/n [7] , taking into account the data of Table 2.

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Table 2. Coefficient X for different soils.

Soil type	Sand	Sandy loams	Clay loam	Clay
Porosity, %	30	25	20	10
X, m/s^2	10^2-10^5	10^{-1}-10^3	10^{-2}-10^1	10^{-3}-10^{-1}

In the plane-radial case, when the pressure depends only on the radius, equation (4) has the form:

$$\frac{\partial p}{\partial t} = \chi\left(\frac{\partial^2 p}{\partial r^2} + \frac{1}{r}\frac{\partial p}{\partial r}\right) \tag{7}$$

We suppose that at r = 0 we have a run-off with flow rate D from an aquifer with thickness *h.* The exact solution (7) for initial and boundary conditions

$$P(r,0)=P_0=h\rho g=const,\ P(\infty,t)=P_0= h\rho g=const,\ D = const,\ t>0 \tag{8}$$

is given in [10].

$$p(r,t) = P_0\left[1 - \frac{D\rho g}{4\pi h P_0 k_f}\int_z^\infty \frac{e^{-\alpha} d\alpha}{\alpha}\right], \tag{9}$$

where z=r2/(4*X*t).

Expression (9) is called the basic formula in the theory of the elastic regime of fluid filtration [13]. It can be represented in the form

$$p(r,t) = P_0\left\{1 - \frac{D\rho g}{4\pi h P_0 k_f}[-E_i(-z)]\right\}, \tag{10}$$

where – Ei(z) – the integral exponential function:

$$-E_i(-z) = -\ln z - 0,5772 - \sum_{n=1}^{\infty}\frac{(-1)^n}{n n!}z^n. \tag{11}$$

The particular derivative of the function (10) along the radius with regard to (11) is equal

$$\frac{\partial p(r,t)}{\partial r}=\frac{D\rho g}{2\pi hk_f r}[1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}z^n]\cdot \tag{12}$$

Substituting expression (12) into formula (2), we obtain the water filtration rate at point *r>0*

$$w=-\frac{D}{2\pi hr}[1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\mathrm{z}^n]\cdot \tag{13}$$

The solution of (10) is true for conditions (8) in the presence of a run-off D at the point r=0.

The initial conditions of the problem will be written in the form

$$P(r,0)=P_0=h\rho g=const,\ P(\infty,t)=P_0=h\rho g=const\ nPu\ r>R. \tag{14}$$

We suppose that on the outer surface of a tube of radius R a flow defined by formula (12) is given, i.e. we have the following boundary condition:

$$\left.\frac{\partial p(r,t)}{\partial r}\right|_{r=R}=\frac{D\rho g}{2\pi hk_f R}[1+\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}b^n], \tag{15}$$

where $b=R^2/4t$.

In this case, the function (10) is a solution of equation (7) with boundary conditions (14) and (15) for a pipe of radius R. In particular, the time dependence of the pressure on the tube surface is determined by the formula

$$p(R,t)=P_0\{1+\frac{\mathrm{D}\rho g}{4\pi hP_0k_f}[\ln b+0{,}5772+\sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\mathrm{b}^n]\}\cdot \tag{16}$$

We represent the expression (16) in the dimensionless form

$$q(a,b)=1+a[\ln b+0{,}5772+\sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\mathrm{b}^n], \tag{17}$$

where

$$q(a,b)=p(R,t)/P_0,\ a=D\rho g/4\pi P_0k_f,\ b=R^2/4\chi t. \tag{18}$$

3 Mechanical impact of water pressure on underground structures

The hydrostatic pressure of water can cause an underground structure float out up, shifting and overturning of the foundation or the appearance of cracks in them. The maximum groundwater pressure is estimated from the formula

$$p=H\rho g,\ \mathrm{Pa}, \tag{19}$$

where H is the head (the height of the water column above the structural element) in m, ρ = 103 kg/m^3 is the density of water, g = 9.81 m/s^2 is the acceleration due to gravity.

The greatest mechanical action on the object, caused by groundwater, takes place in the lower part of the structure, located at the level of the lower boundary of the aquifer [1]. We consider the most unfavorable case, using as an example a foundation in the form of a plate of cylindrical shape of radius R. According to formula (19), the total load on the foundation is

$$F=\pi R2\ H\rho g \tag{20}$$

To calculate the strength and stability of underground structures, special geotechnical software is used. When designing deep foundation pits numerical methods of calculation based on contact models are most widely used. In Russia, for example, programs Wall-3 and PileWallhave been developed. As to foreign software we should note ReWaRD, BMWALL, DEEP, MSheet [10].

We carry out estimated calculations of the stresses caused by groundwaters in the foundation of a circular section of radius R and thickness d. An example of rigid fixing a plate with upper superstructure along the perimeter of its contour is considered in [11]. The maximum tensile load caused by pressure (19) occurs on the surface of the plate near the contour. The corresponding equivalent tensile stress is [11]

$$\sigma_{eq} = \frac{3}{4}\frac{pR^2}{b^2}. \tag{21}$$

An example with an unattached slab differs little from the one considered above - the maximum tensile load under pressure is also in the vicinity of the contour of the slab, while the equivalent tensile stress is [11]

$$\sigma_{eq} = \frac{3}{8}(3+\nu)\frac{pR^2}{d^2}, \tag{22}$$

whereν is Poisson's ratio.

Concrete is usually used as a material for foundations in underground structures. The value of ν of concrete is approximately 0.17. In this case, the coefficient before PR^2/d^2 in expression (22), is 1.19, which is close in the order of magnitude to the analogous coefficient in formula (21) which is equal to 0.75. Therefore, when making calculations as to the strength of the plate, one can use the formula

$$\sigma_{eq} \approx \frac{pR^2}{d^2}. \tag{23}$$

Substituting the expression for the water pressure (16) into formula (19), taking into account (19), we obtain the following expression for determining the equivalent stress

$$\sigma_{eq}(R,t) = \frac{R^2 h\rho g}{d^2}\{1 + a[\ln b + 0{,}5772 + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\mathrm{b}^n]\}, \tag{24}$$

where $a=D/4\pi hk_{\phi}$, $b=R^2/\chi t$.

The tensile strength of concrete σ_b depends on its grade. The allowable value of σ_{eq}, determined by formula (23), should not exceed this value, and the allowable pressure calculated by formula (16) should not exceed p_b, i.e. it should meet the condition

$$p \le p_b = \frac{\sigma_b d^2}{R^2}. \tag{25}$$

In this case, instead of formula (22), the approximate expression is valid

$$\sigma_{eq}(R,t) = \frac{R^2 h\rho g}{d^2}[1 + a(\ln b + 0{,}5772)] \cdot \tag{26}$$

We introduce the dimensionless parameter $c=R^2h\rho g/d^2$, which allows us to determine the equivalent stress σ eq as a function of three dimensionless parameters *a, b* and *c*

$$\sigma_{eq}(a,b,c) = c[1 + a(\ln b + 0{,}5772)] \cdot \tag{27}$$

In the case of the exact formula (24), the equivalent stress is

$$\sigma_{eq}(a,b,c) = c\{1 + a[\ln b + 0{,}5772 + \sum_{n=1}^{\infty}\frac{(-1)^n}{nn!}\mathrm{b}^n]\} \cdot \tag{28}$$

4 Summary

1. Mathematical methods are widely used in various fields of science and technology [1, 14], in particular, in oil industry and in the study of filtration processes in soils [5, 6]. The developed mathematical model allows to study the mechanical action of groundwater on underground structures. Formulas (16), (19) and (24) determine the distribution of water pressure in the construction zone, the pushing force acting on the foundation, and the magnitude of the equivalent stress in the most dangerous place of the cylindrical structure. The size of the aquifer is usually much larger than the radius *R.* In this connection, the formulas obtained can be used to estimate pressures in the case of rectangular-shape structures of with a section of e on f at e≈f. It can be shown that in the first approximation, for this, it is suffice to replace the radius *R* by a/πin expression (16).
2. The formulas discussed contain more than ten parameters, which can be divided into two groups. The first group characterizes the physicochemical properties of the aquifer [20]r. It includes the aquifer filtration coefficient k_f, the coefficients of dynamic μ and kinematic η viscosity, the soil density ρ, the soil porosity *m*, the water volume compression coefficient βw, the coefficient of bulk compression of the hard part of the soil βgr, and the piezo conductivity coefficient χ. The second group includes factors that determine the thickness of the aquifer *h* and the conditions of the external action of the underground structure: time *t*, the rate of water pumping from the aquifer *D*, the size and shape of the underground structure (in the case of a cylindrical shape, radius R).

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